Harmonic Generation based on Harmonicity Weightings

Mauricio Rodriguez
CCRMA & CCARH, Stanford University

A model for automatic generation of harmonic sequences is presented according to the theoretical model of *harmonicity* developed by Barlow (1987). The harmonicity-algorithm is further improved by implementing similarities of interval chord structures and proximity in chord progression by adjacent intervals and/or common tones. The harmonicity-algorithm generates in turn different chord sequences (presented as a recursive tree search engine) that are compared and tested against a target chord progression composed by ear.

Harmonicity

Given an arbitrary (tonal or atonal) chord, how could it be measured the degree of consonance -*harmonicity*- of it? There is a typical sensorial phenomenon that relates simple integer frequency ratios with intervallic relationships perceived as *smooth* or *terse* (Plomp and Levelt, 1965). This observed phenomenon is what usually is called tonal consonance, sensory consonance or harmonic entropy. Following on the rationale of the *simplicity* of numerical relationships as representative ratios for consonant intervals, Clarence Barlow (2003) observed that the *divisibility* of the numbers involved on a frequency ratio is also a crucial attribute for the overall easiness of a heard interval. To measure harmonicity, it would be of use to have a coefficient for natural numbers that would combine their features of simplicity and divisibility. These numeric attributes are expressed in the *indigestibility* function of Eq.1.

$$\xi(N) = 2\sum_{r=1}^{\infty}\left\{\frac{n_r(p_r-1)^2}{p_r}\right\}$$

where

$$N = \prod_{r=1}^{\infty} p_r^{n_r};$$

p is a prime; and
n is a natural number.

(1)

Table 1 Indigestibility values from numbers 1 to 16

N	$\xi(N)$
1	0,000000
2	1,000000
3	2,666667
4	2,000000
5	6,400000
6	3,666667
7	10,285714
8	3,000000
9	5,333333
10	7,400000
11	18,181818
12	4,666667
13	22,153846
14	11,285714
15	9,066667
16	4,000000

Table 1 shows the indigestibility values for numbers 1 to 16. As seen, numbers such as 7, 11 and 13 have a high indigestibility, and therefore, they are more uneasy or complex than obvious simple and divisible numbers such as 2, 4 or 3. From the inversion of the sum of the indigestibility values of the numbers P and Q, a function of *harmonicity* for any interval P:Q can be derived: the more indigestible P and Q, the less harmonic the interval (Eq. 2).

$$h(P,Q) = \frac{\operatorname{sgn}[\xi(P) - \xi(Q)]}{\xi(P) + \xi(Q) - 2\xi(\mathrm{hcf}_{P,Q})}$$

where

$\operatorname{sgn}(x) = -1$ when x is negative, otherwise $\operatorname{sgn}(x) = +1$;
$\mathrm{hcf}_{a,b}$ is the highest common factor of a and b; and
$\xi(x)$ is the indigestibility of x. **(2)**

By measuring the individual harmonicity of each of all the possible interval relations within a chord, it is then taken the geometric mean sum of all of them and that yields a scaled and weighted value that shows the overall harmonicity of any given chord. This *scaled-harmonicity* value is used as the crucial and most important attribute to filter out an automatic generation of chords.

Harmonic generation algorithm

The algorithm for automatic generation of chords receives as main input an abstract map or a *harmonic-abstraction* that shows the direction and voice movement from chord to chord within a harmonic sequence. Fig. 1.

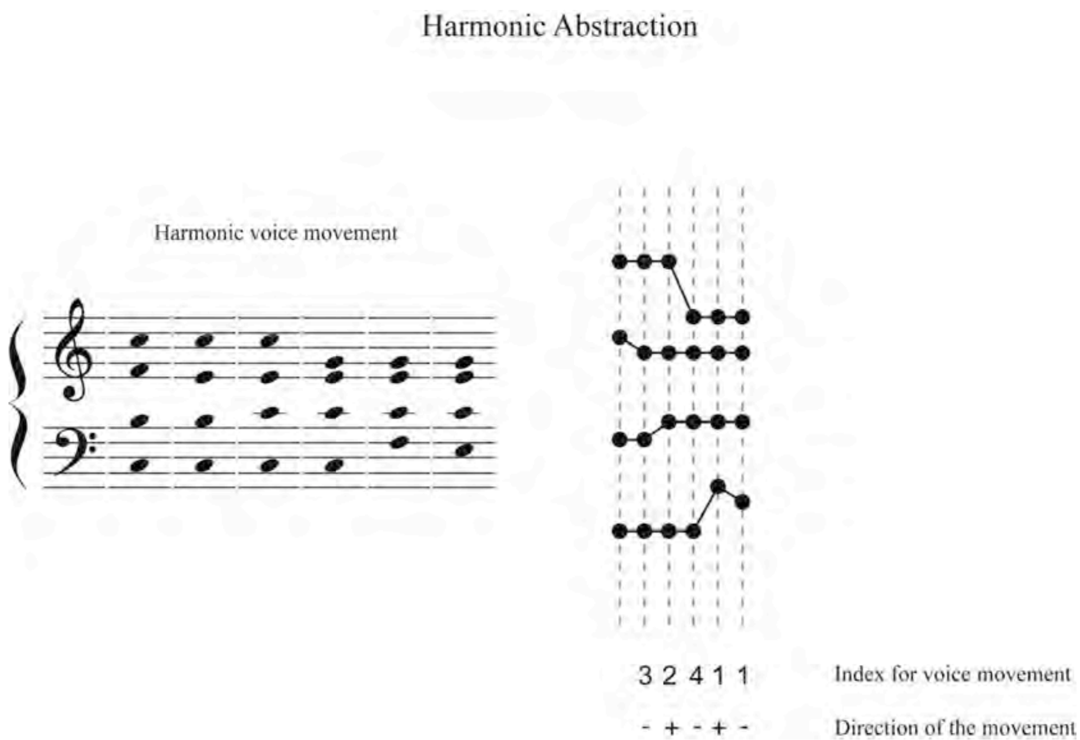

Fig.1.

The harmonic-abstraction is coupled with an "interval reservoir" to which all the interval relations of the new generated chord(s) must observe. Once a set of chords is generated, their scaled harmonicities are sorted keeping the ones that are higher (or lower) in *consonance* regarding to a "chord-seed" and a consonance/dissonance profile. To improve the performance of the harmonic-generator, chords presenting adjacent and/or common tones and having a similar interval structure in relation the chord-seed are ranked with a higher score; the chords maximizing in the final score are the best candidates to follow the sequence (Table 2).

Table 2 Sample to show the ranking of generated chords

```
Harmonic-entity | Harmonicity (>) | Common-interval-structure | Common-notes | Adjacent-degree (weighted) ||Score||
(in MIDI values)  (less harmonic
                  chords)


((59 61 74 76) 0.08302280901376337 (1 3 5) (59 76) 3)                                                          = 8
((49 63 66 76) 0.07907755450048336 (1 3 5) (63 76) 6)                                                          = 11
((59 60 73 76) 0.07851176005378097 (1 3 4 4 5) (59 76) 3)                                                      = 10
((59 63 74 76) 0.07714315091429033 (1 3 4 5 11) (59 63 76) 3)                                                   = 11
((59 72 73 76) 0.07606288989834777 (1 3 4 4 5) (59 76) 2))                                                      = 9


Chords (49 63 66 76) and (59 63 74 76) are the best possible continuation of the chord-seed: (48 59 63 76)
```

Harmonic tree recursion

To test every possible chord sequence that can be generated with the constraints imposed by the harmonic abstraction and the consonance/dissonance profile, a recursive tree algorithm searches all the suitable candidates from one generated chord to another until a desired chord-sequence length is reached. Once the search is finished, all the sequences satisfying the desired length are played one after the other to test their musical relevance. Fig. 2.

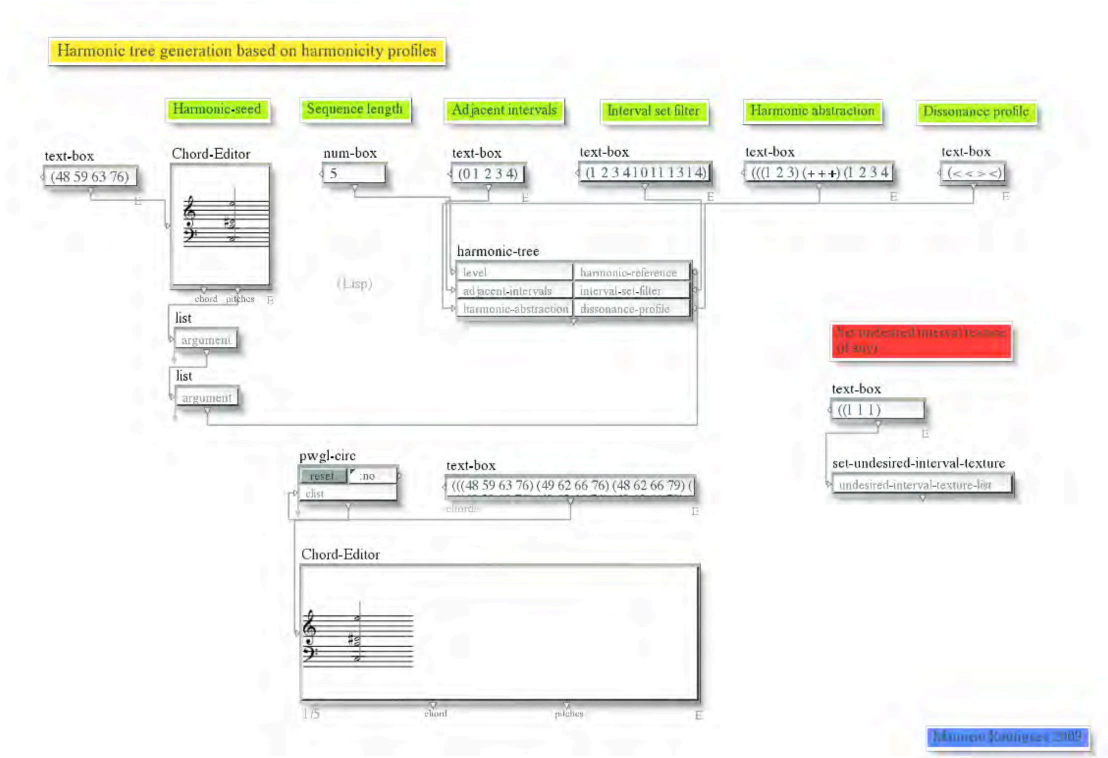

Fig.2.

Algorithm test

To evaluate the performance of the harmonic-tree algorithm, a manually (aurally) composed chord sequence of five chords is compared with all possible results of an equivalent automatic-generated sequence. Both, composed and generated sequences are based in the following harmonic abstraction and dissonance profiles (Fig. 3.):

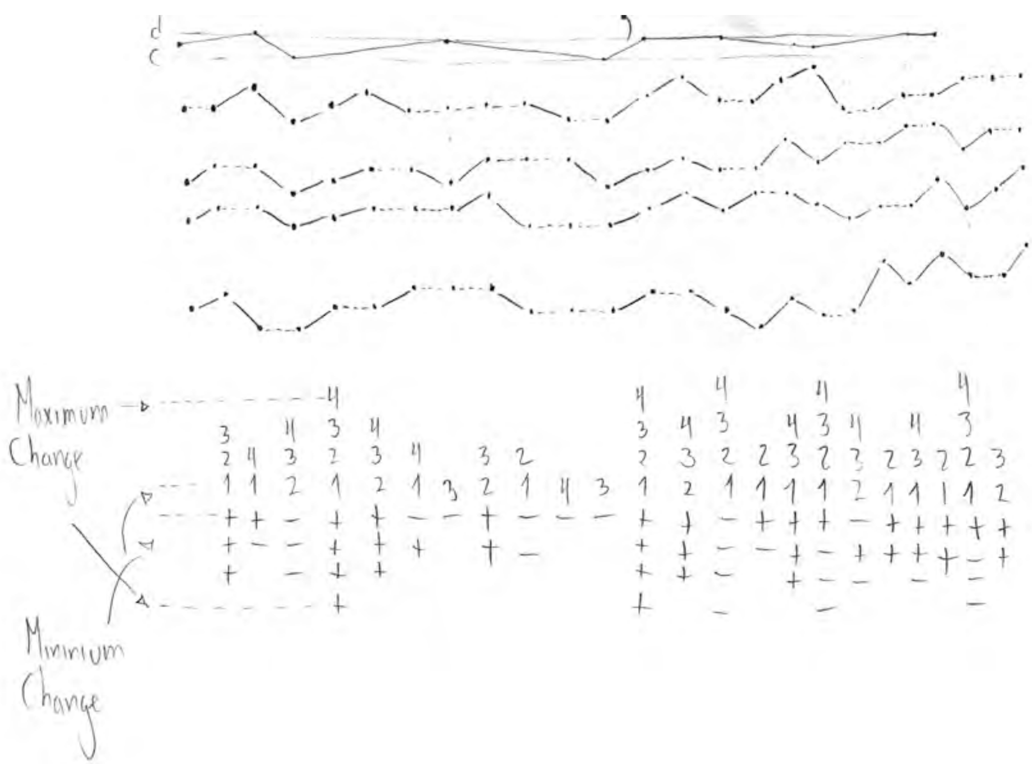

Fig. 3. D and C on top part of the figure stand for Dissonance and Consonance respectively.

Fig.4. shows the automatically generated results. At the top, the enclosed grey sequence presents the originally composed (target) harmonic sequence as it appears in the first pages of my trio piece entitled *//ligero//* (Figs. 5. and 6.); the bottom enclosed grey sequence is chosen as the best result for presenting strong similarities with the original sequence. Each of the 9 automatically generated sequences is consistent with the harmonic abstraction and the consonance/dissonance profiles; however, chord sequence number 8, captures the soprano melodic line in its totality and the similarities with the bass line are reasonable good.

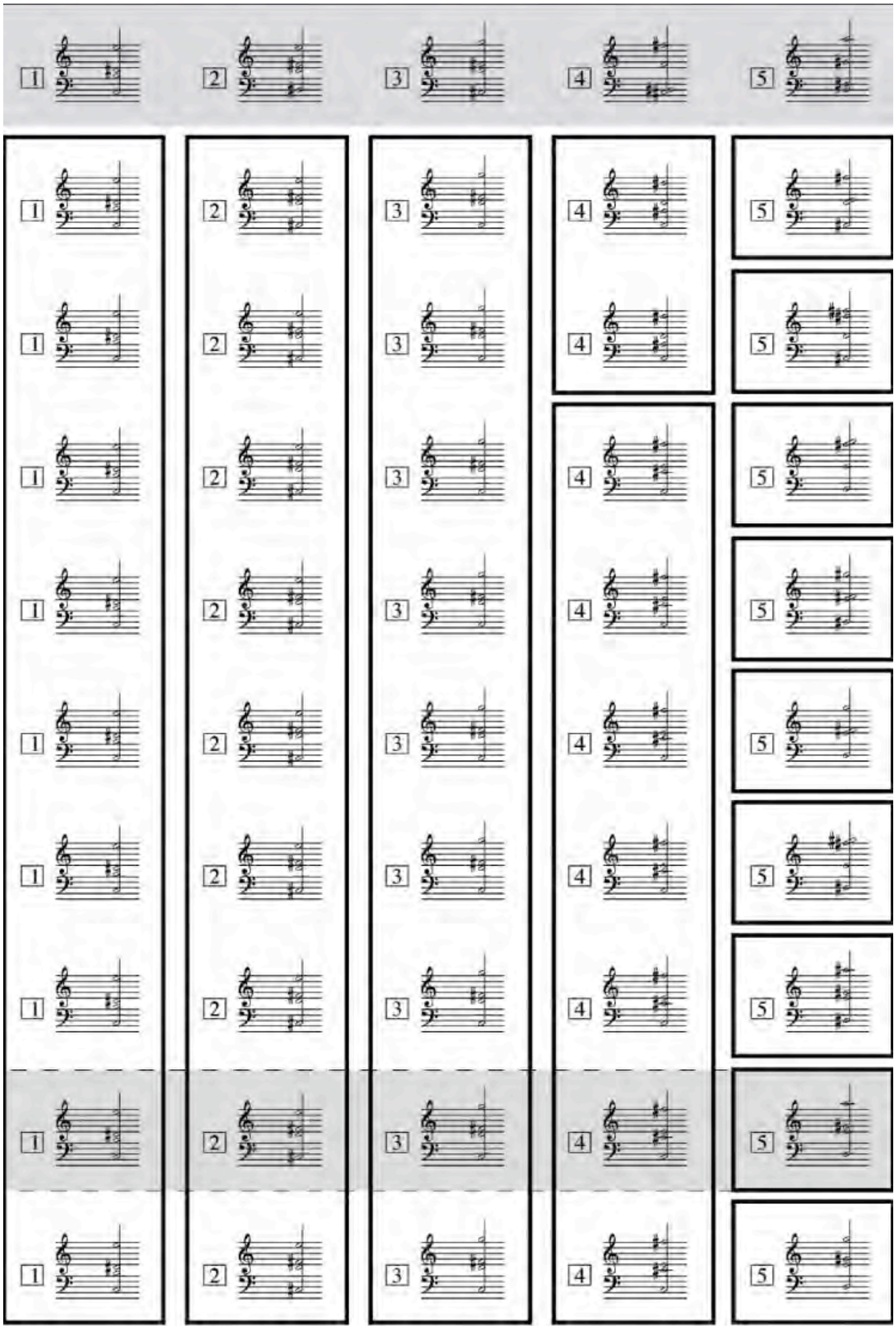

Fig. 4.

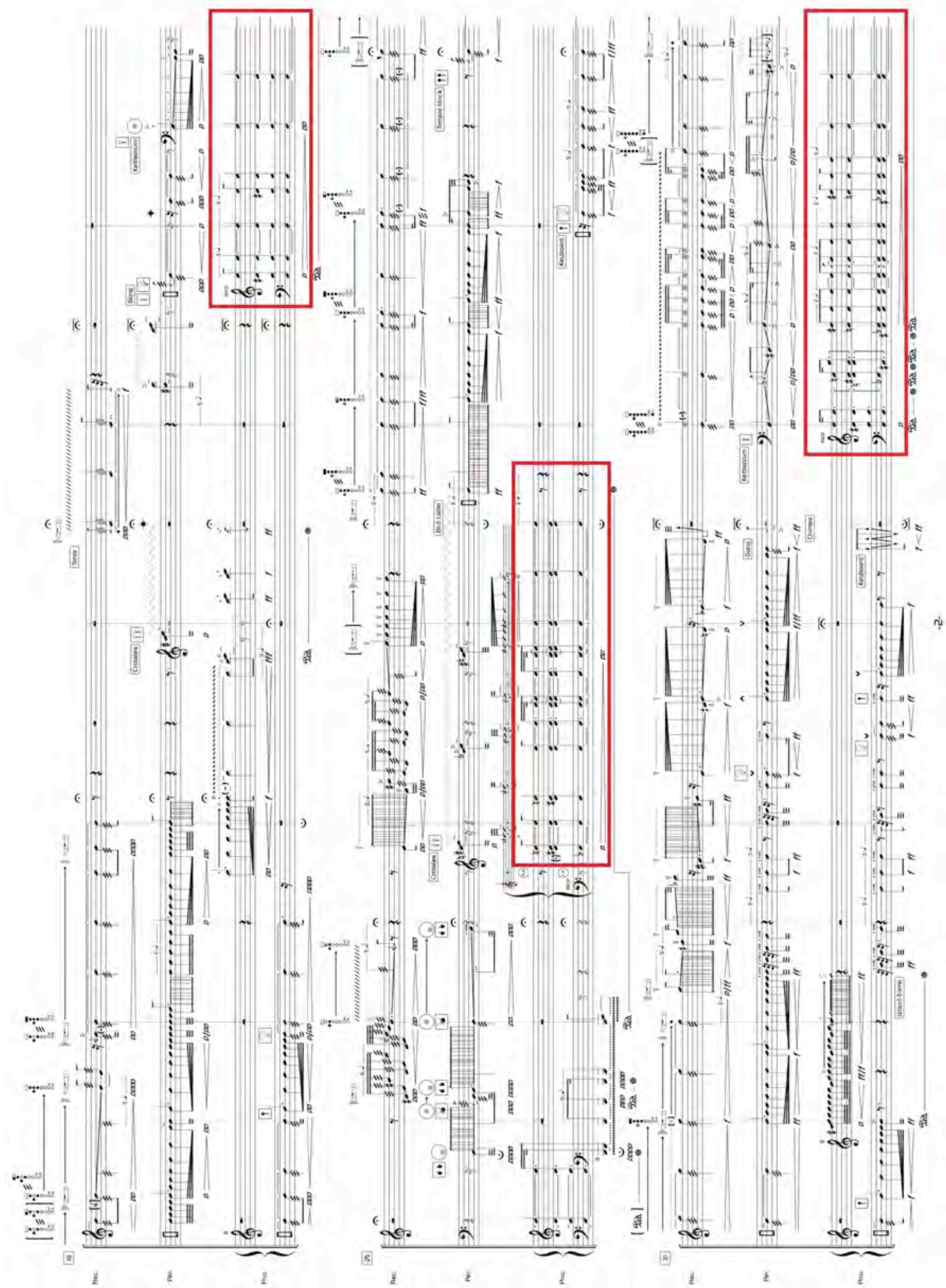

Fig. 5.

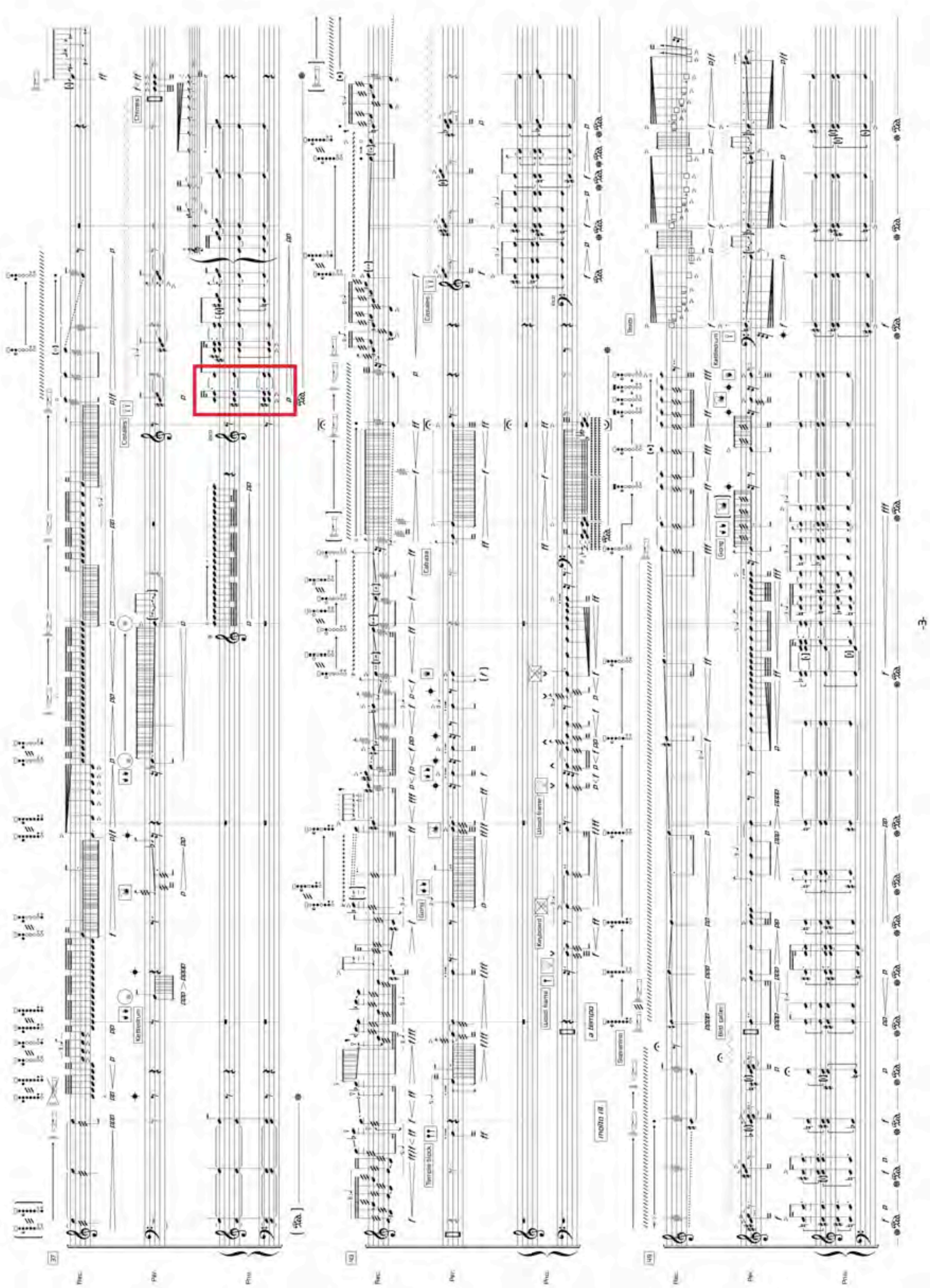

Fig. 6.

Further improvements

Since the dissonance/consonance profile is perhaps the most important attribute to filter out the possible paths from one chord generation to another, it will be of a great use to have more refined dissonance-consonance ranks or levels that could show with more accuracy the different nuances of harmonic tension (or distension). The other attributes that improved the harmonic-generation, such as the adjacent and/or common tones or the similarities in interval structure, have indeed proven their usefulness to yield sequences with highly convincing musicality; however, other attributes such as the textural balance of the chord, pitch-class preferences, the *effort-ness* in the melodic voice leading of independent voices, or other possible attributes that help to the construction of well formed harmonic progressions, remain unexplored. These and other possible attributes to be implemented in the algorithm here described, could be evaluated under the light of a more critical revision of the rules governing the "always-fresh" and wise principles of traditional harmonic theory.

Bibliography

Barlow, C. "Two Essays on Theory," Computer Music Journal. Vol. 11, No. 1, 44-60 (1987).

Barlow, C. *On Musiquantics*, Royal Conservatory The Hague. (2003).

Plomp, R. and Levelt, W. J. M. "Tonal Consonance and Critical Bandwidth," Journal of the Acoustic Society of America. Vol. 38, 548-560 (1965).

Sethares, W. A. *Tuning, Timbre. Spectrum, Scale*, Springer-Verlag, London (2005).